DUE DATE:______________ NAME ________________

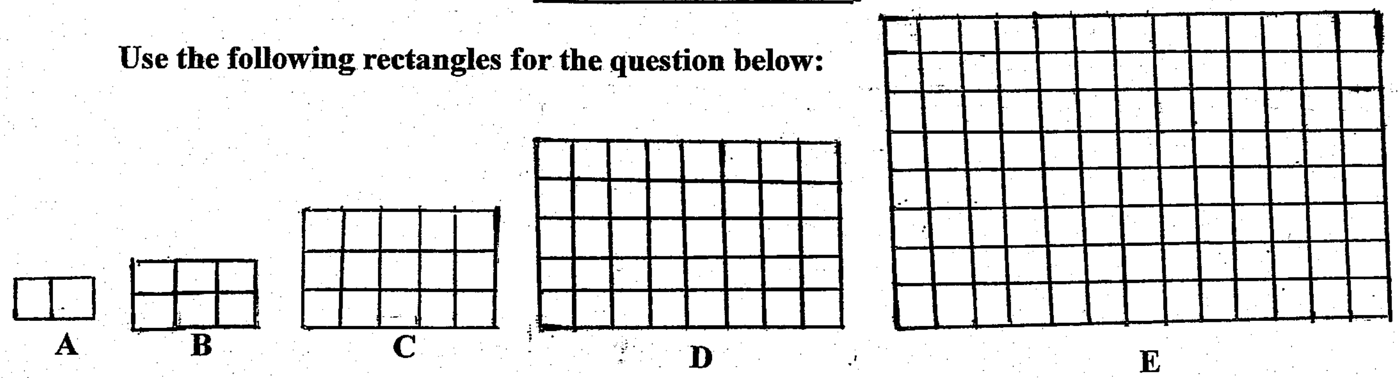
DATE ___________

PERIOD ___

52 points
+ 5 points for neatness, effort

PRE-ALGEBRA ACCELERATED – 8.2
THE GOLDEN RATIO

Use the following rectangles for the question below:



1. Find the length-to-width ratio of each rectangle above. Make each ratio a decimal. Then find the width-to-length ratio of each rectangle and make each ratio a decimal.

A= **B=**

C= **D=**

E=

2. Describe the pattern of the lengths and widths. Describe the difference between length-to-width and width-to-length. Predict the lengths and widths and write them in both ways (L to W and W to L) of the next three rectangles in the pattern.

F= **G=**

H=

<u>Pattern Description</u>:

Now put your results from above in the following table (fill in all the empty boxes) :

LENGTH	**2**	**3**	**5**	**8**	**13**			
WIDTH	**1**	**2**	**3**	**5**	**8**			
RATIO								
DECIMAL								
WIDTH	**1**	**2**	**3**	**5**	**8**			
LENGTH	**2**	**3**	**5**	**8**	**13**			
RATIO								
DECIMAL								

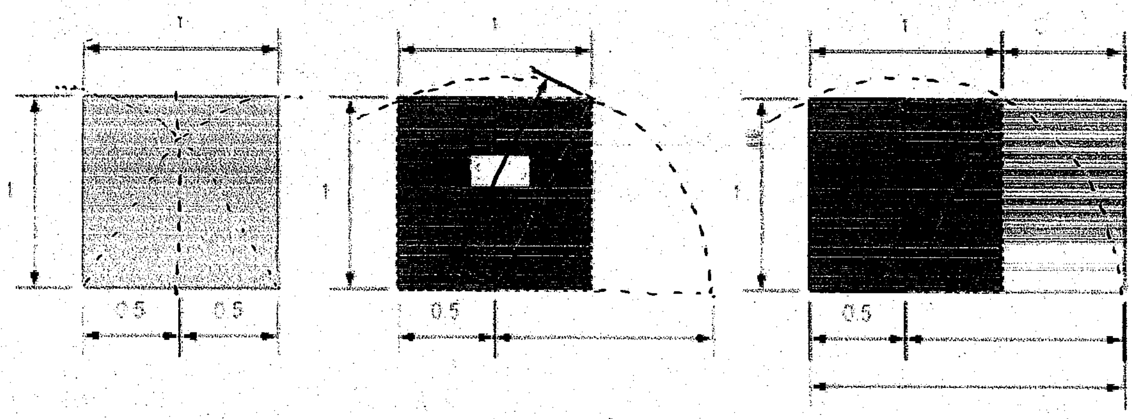
3. The *golden ratio* is the number $\frac{\sqrt{5}+1}{2}$. This is called *Phi.* Use a calculator to round this number to three decimal places. How does the result relate to the table above? (Remember: look carefully where the square root symbol – it does not go all the way across the numerator!)

4. Now you construct a golden rectangle following the steps on the next page and label your page <u>Appendix A: Golden Rectangle</u>. Measure its length and width to see how close your rectangle is to a golden rectangle. Write a sentence below stating how close you were.

Follow these steps to draw your own golden section using a straight edge and a compass:

1. **Start with a perfect square having a length of one unit on each side.**
2. **Bisect the square vertically.**
3. **Draw a diagonal line from the baseline midpoint to the upper right corner of the square.**
4. **Strike an arc from this diagonal line, using the baseline midpoint as the center of the arc.**
5. **Extend the baseline of the square out to the right until it meets the arc.**
6. **Draw a vertical line upward from the baseline extension.**
7. **Extend the top line of the square out to the right until it intersects the vertical line.**

Congratulations! You have created a golden rectangle.



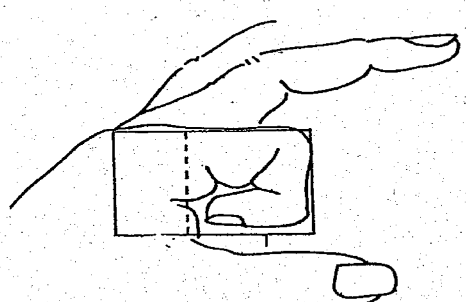
5. Make a copy of your rectangle. Tear the original square from the copy. Measure the length and width of the rectangle that remains. What can you conclude? Label these parts <u>Appendix B: More Golden Rectangle</u>.

6. Some artists and architects have created designs based on the *golden rectangle*. People seem to find the rectangle's proportions pleasing to the eye.

Find a *golden rectangle* in the photo of the Parthenon below. Outline it in red. Then find a picture of another building that has a golden rectangle. Include this as <u>Appendix C: Golden Rectangle in Architecture</u>. (You can use the computer or any reference book.) Make sure to outline in red any golden rectangles in the structure you find.

7 You have your own personal *Golden Ratio*. Fold your left index finger in on itself. The knuckle on your hand to the middle knuckle form the length of a rectangle (y), and between the middle and top knuckle the width of the rectangle is formed (x).

. Measure lengths x and y. Find the decimal form of $\frac{x}{y}$.



8. Now, choose three of the attached activities to practice the *Golden Ratio*. Submit your activities as <u>Appendix D: *name of your activity*</u>, <u>Appendix E: *name of your activity*</u>, *and* <u>Appendix F: *name of your activity*</u>. Some of the activities can be completed on the papers right in the packet – just label them at the top.

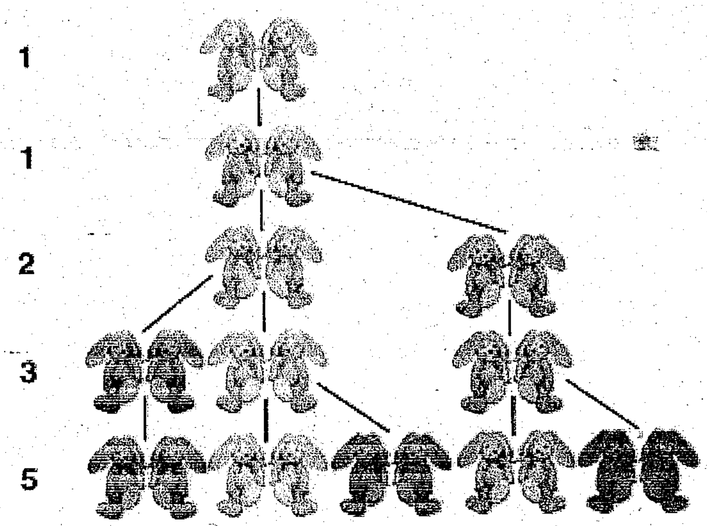
Activity 1: FIBONACCI

Fibonacci used his sequence of numbers to investigate the population growth of his favorite furry lop-eared friend, the rabbit. He based his model on a maximum-security bunny heaven where rabbits cannot escape or die, and the problem he devised goes like this...

Suppose a newly born pair of rabbits (one male and one female) are put in a field. These rabbits take a month to become sexually mature, after which time they produce a new pair of baby rabbits or 'kits' (again, one male and one female). How many pairs will there be in subsequent years?

Think about it and if your answer is "enough for a very large pie", think some more.

Number of pairs per month



As models of population growth go, it may not be the best, as it does not allow for rabbits to have more than two kits, to have kits of the same sex, or to take time off at Easter to deliver goodies. However, as a special series of numbers, the Fibonacci sequence has a hidden beauty all of it's own. Count the number of florets spiralling out from the centre of a cauliflower. Look closely and you will find two spirals running in opposite directions, and the number of florets in each are two consecutive Fibonacci numbers.

Raising Rabbits

Begin with one adult pair of rabbits in a pen in January.

Let them multiply according to these rules.

1. Each month, all *A*dult pairs have a *B*aby pair.
2. All old *B*aby pairs simply become *A*dults after a month. *(Baby pairs do not have babies until after they become adult pairs!)*

Complete this chart showing the number of rabbits in the pen each month. Assume none die.

	***B*aby pairs**	***A*dult pairs**	**Total pairs**
Jan.	0	1	1
Feb.	1	1	2
Mar.	____	____	____
Apr.	____	____	____
May	____	____	____
June	____	____	____
July	____	____	____
Aug.	____	____	____
Sept.	____	____	____
Oct.	____	____	____
Nov.	____	____	____
Dec.	____	____	____

1. What pattern can you identify here?

2. How many rabbits would you expect to find in June of the following year?______
3. When will there be 1 million rabbits in the pen?______
4. There were two men, of whom the first had 3 small loaves of bread and the other 2; they walked to a spring where they sat down and ate. A soldier joined them and shared their meal, each of the three men eating the same amount. When all the bread was eaten, the soldier departed, leaving 5 bezants to pay for his meal. The first man accepted 3 of the bezants, since he had had 3 loaves; the other took the remaining 2 bezants for his two loaves. Was the division fair?

Fibonacci Tree

Binary numerals are constructed in the same manner as decimal numerals, with two important exceptions:

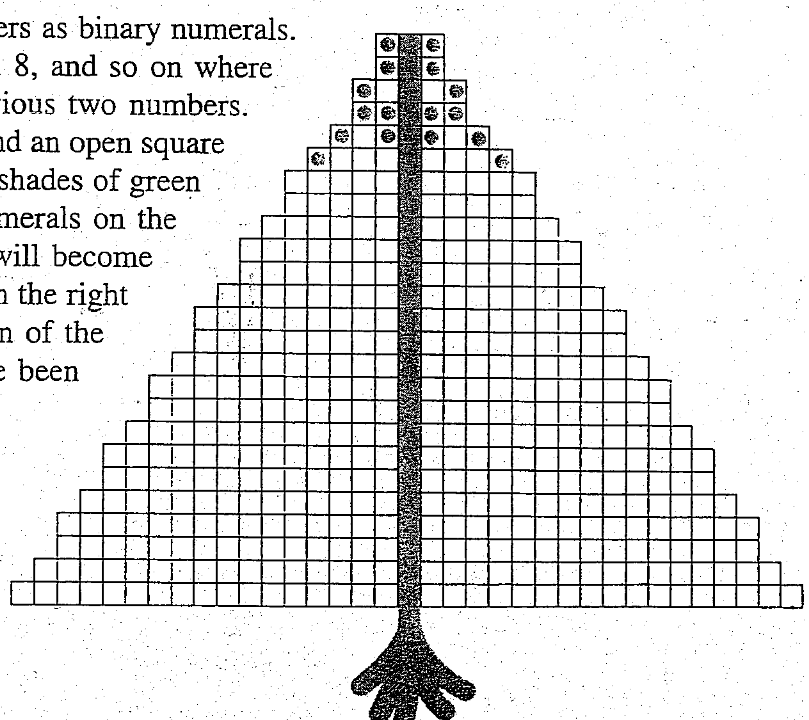
- Each successive place value is two times rather than ten times the previous value.
- There are only two symbols (0, 1) rather than ten (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Decimal	8s	4s	2s	1s	Binary
1				1	1
2			1	0	10
3			1	1	11
4		1	0	0	100
5		1	0	1	101

For example: The number thirteen can be written both as a decimal numeral and as a binary numeral:

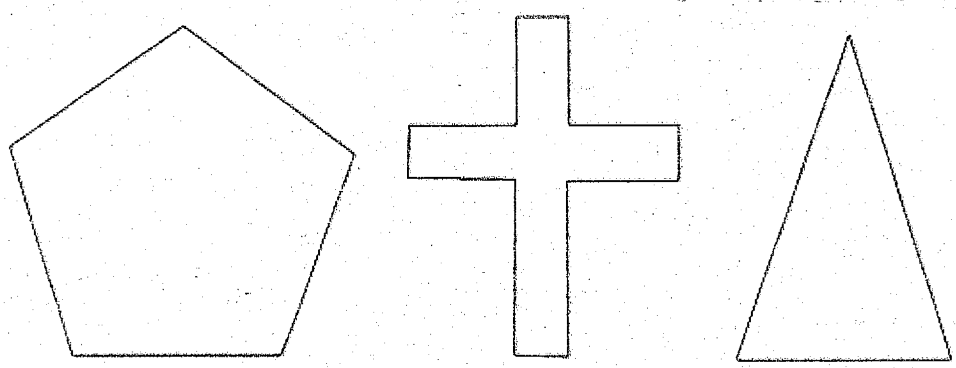
	10 1	(place values)
Decimal	1 3	because $(1 \times 10) + (3 \times 1)$ is thirteen
	8 4 2 1	(place values)
Binary	1 1 0 1	because $(1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$ is thirteen

Express the first 25 Fibonacci numbers as binary numerals. Fibonacci numbers are 1, 1. 2, 3, 5, 8, and so on where each number is the sum of the previous two numbers. Using a solid square to represent 1 and an open square to represent 0 (or different colors or shades of green for the two symbols), record the numerals on the left half of this tree. The numerals will become larger as you move down the tree. On the right side of the trunk, show the reflection of the left side. The first six numerals have been done for you.



Activity 2: The Golden Ratio and Geometry

The Golden Ratio



You can do the following explorations by using a protractor to draw on paper, or by using Geometer's Sketchpad.

REGULAR PENTAGONS

Draw a regular pentagon (to get you started recall that the interior angles have measure 108 degrees) and also draw in one diagonal of the pentagon. Measure the length of one side of the pentagon and measure the length of the diagonal. What is the ratio of the side to the diagonal?

GRAVEYARD CROSSES

A German psychologist by the name of Gustav Fechner studied the crosses in graveyards and discovered an interesting fact about their dimensions. Measure the upper and lower portions of the main stem of the cross which is cut by the crossbar. What is the ratio of the lengths of these two portions?

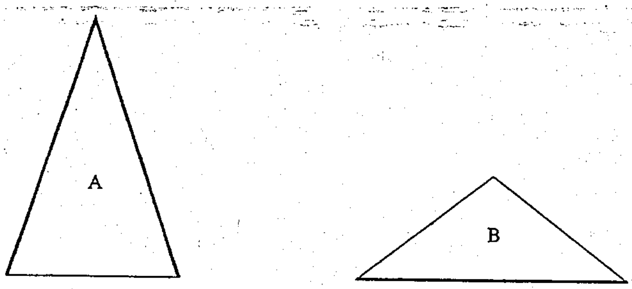
ISOSCELES TRIANGLE

Draw an isosceles triangle with base angles equal to 72 degrees. Measure the length of the shorter side and the two legs, which of course have the same length since this is an isosceles triangle. What is the ratio of the lengths?

Now rotate the shorter side through the base angle until it touches one of the legs. From the endpoint draw a segment down to the opposite base vertex. The original isosceles triangle is now split into two triangles. Calculate the area of the two smaller triangles. What is the ratio of the areas?

Tantalizing Triangles

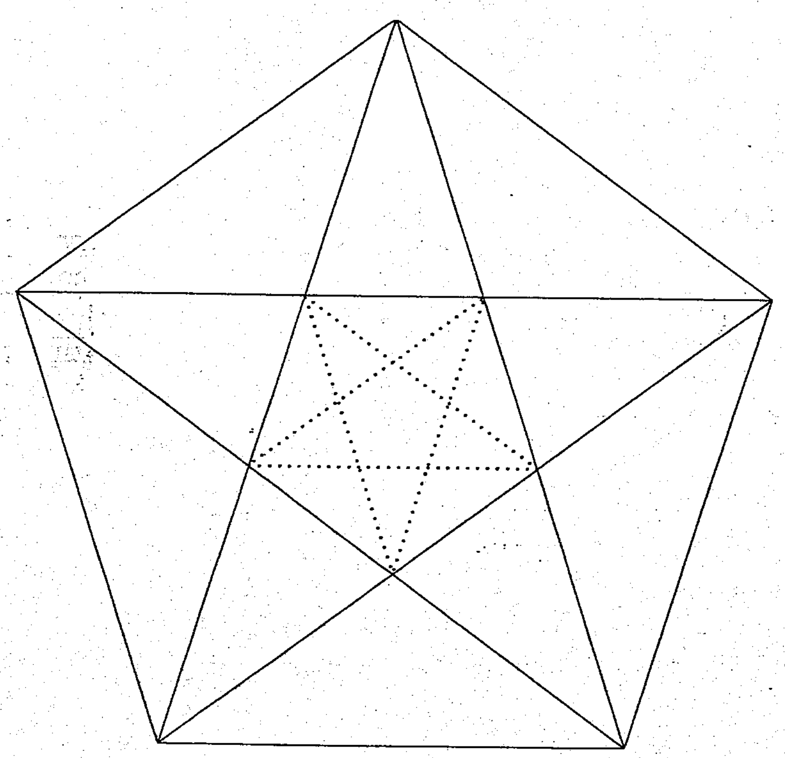
Two kinds of triangles have golden proportions. In both cases, the ratio of the short side to the long side is the same—the golden ratio, 0.618. . . to 1.



Shape *A* has two long sides and one short side.

Shape *B* has two short sides and one long side.

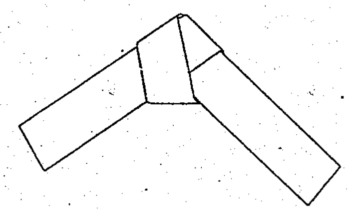
1. In the large, solid-lined figure, how many shape *A* golden triangles are there of any size? _____
2. In the large, solid-lined figure, how many shape *B* golden triangles are there of any size? _____
3. Including the dotted-lined figure, how many golden triangles of either shape and of any size can you find? _____

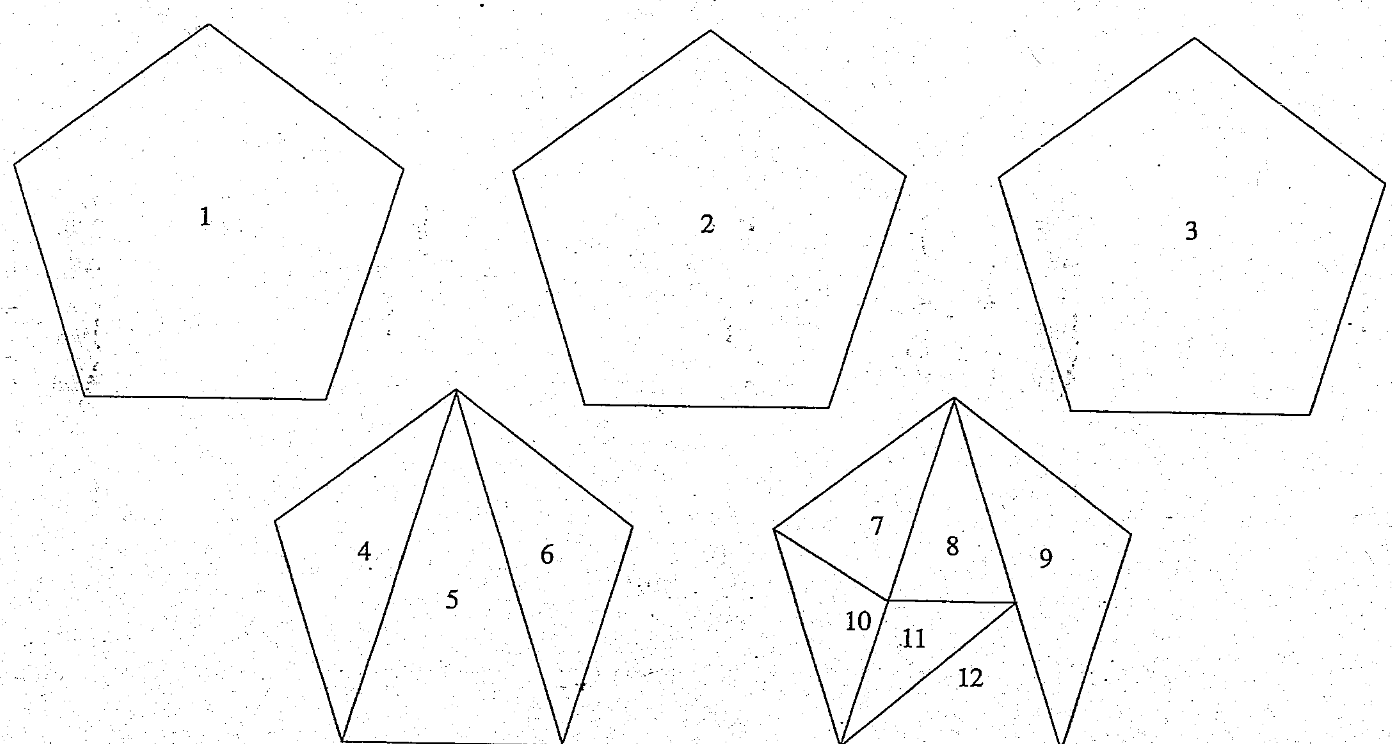


Pentagon Play

Cut out the five pentagons at the bottom of the sheet. Cut two of them into the additional pieces as marked. You will then have three pentagons and nine golden isosceles triangles. They are golden because the ratio of the short sides to the long sides is 0.618 to 1. They are isosceles because two sides are the same.

1. How many of the triangles have two short sides and one long side? ______
2. How many different sizes are there of these short-sided triangles? ______
3. How many of the triangles have two long sides and one short side? ______
4. How many different sizes are there of these long-sided triangles? ______
5. Fit all 12 pieces together into one large pentagon.
6. Cut a strip of paper 1 in. wide and about 11 in. long. Fold it into a flat knot—very gently! Can you get a perfect pentagon? Find the golden triangles in your folded paper.





Activity 3: Golden Ratio: A Golden Greek Face

A GOLDEN GREEK FACE

Toolbox: Calculator; metric ruler (measures to mm)

Statues of human bodies considered most perfect by the Greeks had many Golden Ratios. It turns out that the "perfect" (to the Greeks) human face has a whole flock of Golden Ratios as well.

You'll be measuring lengths on the face of a famous Greek statue (with broken nose) by using the instructions on *this* page. Before you start, notice that near the face on the second page are **names** for either a **location** on the face or a **length between** two places on the face. Lines mark those lengths or locations exactly.

Using your cm/mm ruler and the face picture on the next page, find each measurement below to the nearest millimeter, that is tenth of a cm or .1cm (___._ cm). Remember, you are measuring the distance or length between the two locations mentioned. You can use the marking lines to place the ruler for your measurements. Fill in this table.

a = Top-of-head to chin = ___.__ cm
b = Top-of-head to pupil = ___.__ cm
c = Pupil to nosetip = ___.__ cm
d = Pupil to lip = ___.__ cm
e = Width of nose = ___.__ cm
f = Outside distance between eyes = ___.__ cm
g = Width of head = ___.__ cm
h = Hairline to pupil = ___.__ cm
i = Nosetip to chin = ___.__ cm
j = Lips to chin = ___.__ cm
k = Length of lips = ___.__ cm
l = Nosetip to lips = ___.__ cm

Now use these letters and go on to the next page to compute ratios with them. Use your calculator!

Now, find these ratios to three decimal places, using your calculator:

$$\frac{a}{g} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

$$\frac{b}{d} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

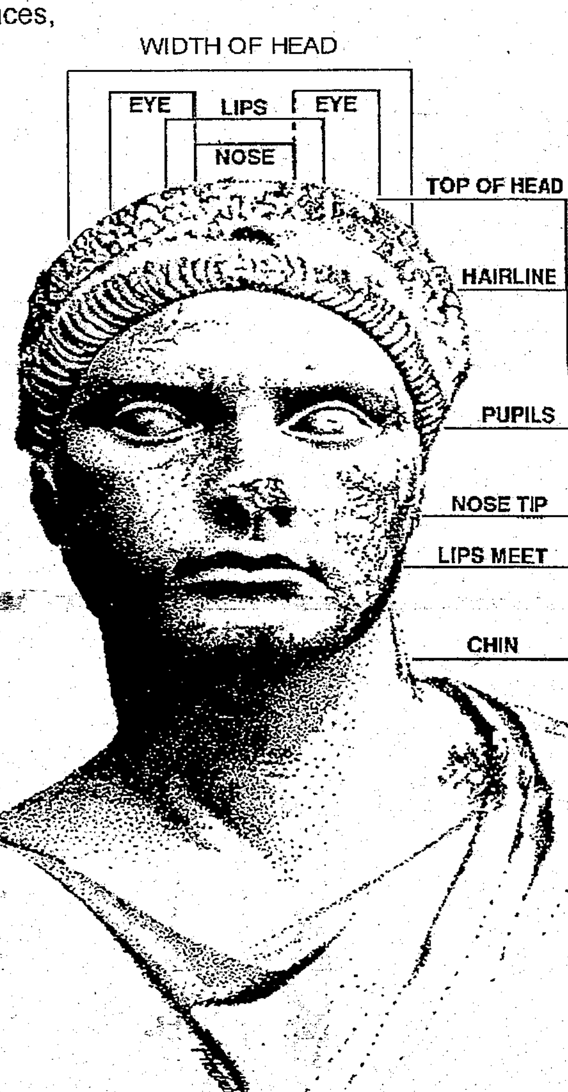
$$\frac{i}{j} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

$$\frac{i}{c} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

$$\frac{e}{l} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

$$\frac{f}{h} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$

$$\frac{k}{e} = \frac{\_\_\_\_\ \text{cm}}{\_\_\_\_\ \text{cm}} = \_\_\_\_\_\_$$



Activity 4: Golden Ratio of the Human Body

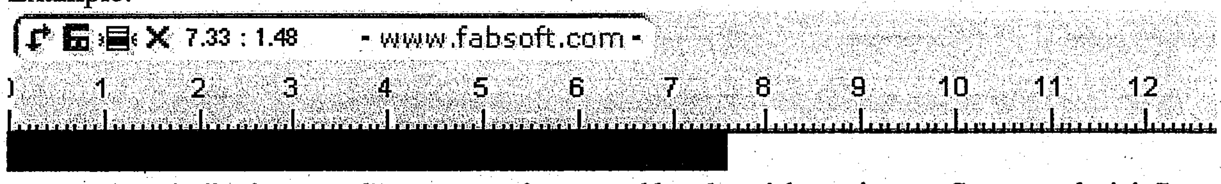
(need a partner)

I. Review of Length Measurement

Measure the length of each line in centimeters. Record your answers to *2 decimal places* and to *1 decimal place*. The following steps will help you to determine the length of each line.

A. Determine the whole-number length of the line.
B. Count the number of short hash marks to the right of the whole number. Each hash mark is 0.1 of a centimeter.
C. Estimate the final digit.

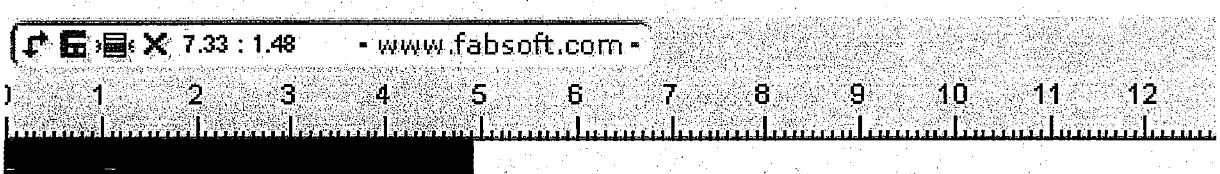
Example:



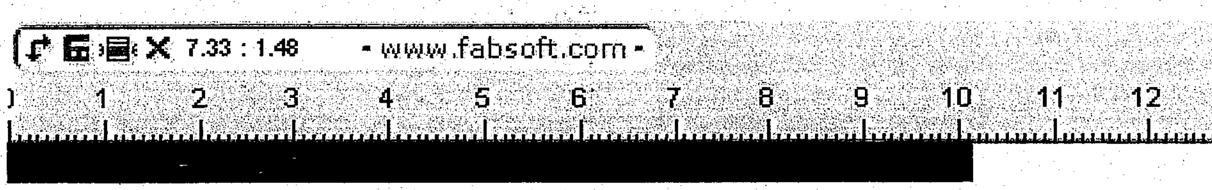
A. The line is greater than seven centimeters and less than eight centimeter. Consequently, it is 7 **whole centimeters** plus a fraction of a centimeter.

B. The line ends between the 5th and the 6th small hash mark to the right of the 7th whole centimeter. Each hash mark is 0.1 cm, or 1/10, of a centimeter. Consequently, the length of the line is **between 7.5 cm and 7.6 cm.** (cm is the abbreviation for centimeter.)

C. The length of the line appears to be closer to 7.6 cm than 7.5 cm. Consequently, the length of the line may be stated as **7.59 cm**, <u>but the last digit is a guess</u>. If one rounds this answer to 1 decimal place, then the answer is **7.6 cm.**



1. a. length of line (2 decimals) _______ cm b. length of line (1 decimal) _______ cm



2. a. length of line (2 decimals) _______ cm b. length of line (1 decimal) _______ cm

II. Total Body Height to Navel Height

The purpose of this activity is to determine the ratio of total body height to navel height. Read the instructions carefully. Work in pairs. Each person must complete one of these data sheets.

1. Obtain two meter sticks and 3 Post-it Notes.

2. One partner will determine the total body height and navel height of the partner.

3. *Partner being measured:* Take off your shoes (highly recommended). Stand upright with your back against a wall. Put your feet together.

4. *Partner taking measurements:* Place the meter stick on top of your partner's head to help you determine your partner's height. Make sure the meter stick is level with the floor. Mark your partner's height on the wall with a Post-it Note. Determine your partner's total body height by measuring the distance from the floor to the Post-it Note. (Use a second Post-it Note for the 1-meter mark.) Record your partner's name in space 1 and his/her total body height (to one decimal place) in space 2.

5. *Partner being measured:* Continue standing upright. Face the wall. Point to your navel with a pen/pencil. Keep the pen/pencil horizontal with the floor and mark your navel height with the third Post-it Note.

6. *Partner taking measurements:* Measure the distance from the floor to the Post-it Note indicating your partner's navel height. Record the height of your partner's navel in space 3 to one decimal place.

7. Trade roles and repeat steps 1-6. Fill in spaces 4-6.

Table 1: Total Body Height/Navel Height

Name of Partner	Total Body Height	Navel Height	Ratio of Total Body Height / Navel Height	Ratio of Total Body Height/Navel Height in Decimal Form
1.	2. cm	3. cm	____ cm / cm	
4.	5. cm	6. cm	____ cm / cm	

8. *Total Body Height/Navel Height* is the ratio of total body height to navel height. Complete the *Total Body Height/Navel Height* column in the table by filling in the body heights and navel heights.

9. Using your calculator, divide *Total Body Height* by *Navel Height* and report your answer to 3 decimal places.

B. Reciprocals

Two numbers whose product is 1 are called reciprocals.

For example: $\frac{3}{2} \times \frac{2}{3} = 1$

2/3 and 3/2 are reciprocals.

Sometimes, the Golden Ratio is expressed as its reciprocal. (1) Using the approximation of the Golden Rule determine the reciprocal of the Golden Ratio. (2) What is the reciprocal of your Total Body Height/Navel Height?

(1)

(2)

Activity 5: Golden Ratio Around You!

Which things around you are made in the golden ratio?

Measure the following items with a centimeter measuring tape or stick. (Round measurements to the nearest .5 cm.) Then rank the items from closest (1) to the golden ratio to least close (10).

RANK	ITEM MEASURED	LENGTH (cm)	WIDTH (cm)	RATIO (L:W)
	TV Screen			
	Calculator			
	Math Textbook			
	Notebook Paper			
	$1 Bill			
	3" x 5" index card			
	Your school desk top			
	Other:			
	Other:			
	Other:			

Find an item that has a ratio even closer to the golden ratio. List its dimensions and ratio below.

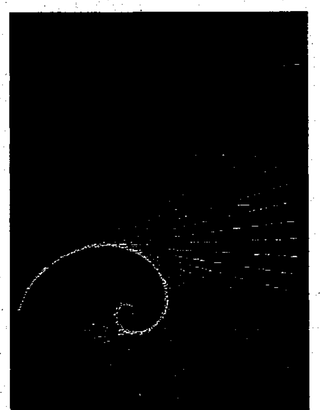
Item Measured	Length(cm)	Width(cm)	Ratio(L:W)

1. What are some **logos** you can find that are golden rectangles? Either draw them or cut them out of a magazine or newspaper or you may get them from the Internet.
2. Where else have you found the golden rectangle? Give at least 2 examples.

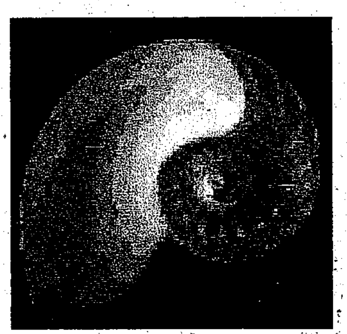
Activity 6: Create a Golden Spiral

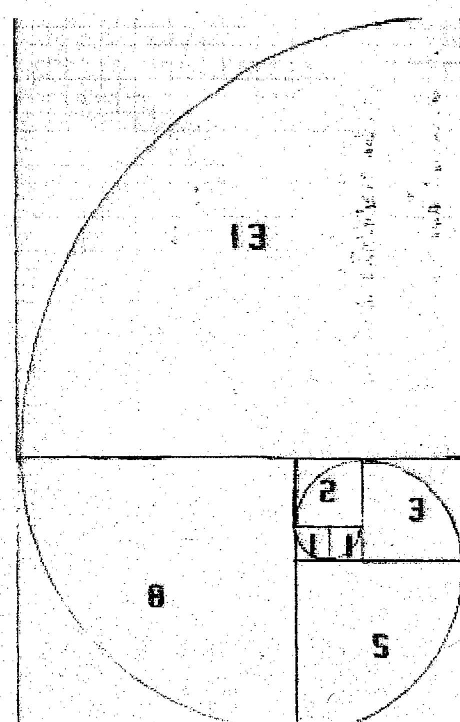
The spiral is a beautiful mathematical curve that we often see around us. The picture to the right shows a piece of ornamental metalwork, a bracket for a sculpture.

The spiral below is a beautiful piece of geometric artwork:



The chambered nautilus seashell is in the shape of a spiral. The creature who lives in the shell has built the chambers of the shell as it grew. The spiral is a curve that often is related to growth outward from the center.

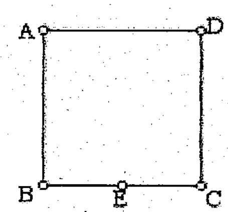




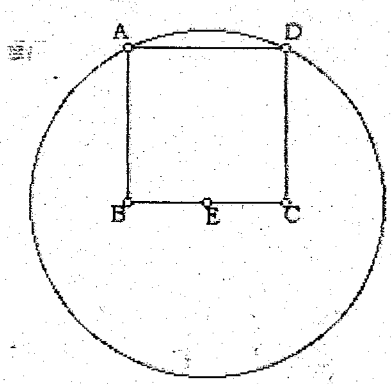
The Golden Spiral above is created by making adjacent squares of Fibonacci dimensions (I did mine using a scale of 2). An arc is then made across each square.

You can construct a mathematical spiral, by following these steps: (you may want to use graph paper.)

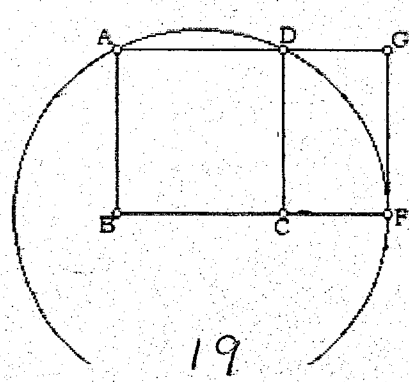
Step 1: Construct a square (ABCD) and label the midpoint of one side (point E).



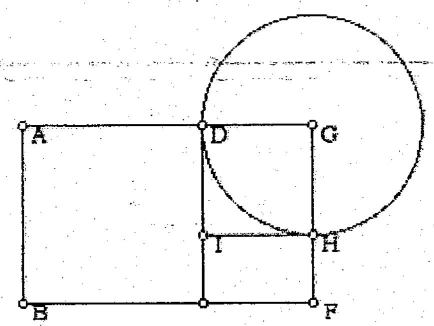
Step 2: Using that midpoint (E) as a center, construct a circle passing through the opposite corner of the square (point A).



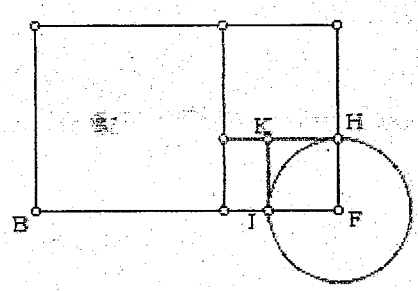
Step 3: extend side BC until it intersects the circle (at point F). Construct a rectangle (ABFG). this rectangle is called the Golden Rectangle, and the ancient Greeks believed that it's shape was the most pleasing shaped rectangle to the eye, and they called the ratio between the length and width of this rectangle The Golden Ratio.



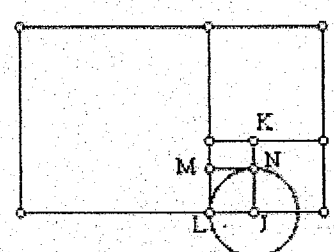
Step 4: Construct a circle with center G and passing through point D. Where this new circle intersects segment GF is point H. Construct a square DGHI.



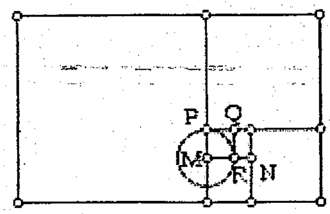
Step 5: Construct a circle with center F and passing through point H. Where this new circle intersects segment FB is point J. Construct a square HFJK. Notice that this is the same process as in step 4.



Step 6: Construct a circle with center J and passing through point L. Where this new circle intersects segment JK is point N. Construct a square JLMN. Notice that this is, again, the same process as in step 4 and in step 5.



Step 7: Repeat the process: construct a circle with center M and passing through point P. Where this new circle intersects segment MN is point R. Construct a square MPQR.



Step 8: Construct arcs centered at the corners of the squares, and inscribed in the consecutive squares as shown. This is not a true spiral, as it is composed of quarter-circles, but it is still called The Golden Spiral. The ancient Greeks believed that the ratio of the sides of this rectangle were the most beautiful of ratios, and they called this The Golden Rectangle. Color your spiral and make it look beautiful.